Math 213 Calculus III

Spring 2011
Tuesday, May 10
Sections 10.3-10.5
Topics:

1. The arc length and curvature formulas
2. The independence of arc length and parametrization
3. The geometric definition of curvature
4. The TNB frame
5. Definitions of velocity and acceleration as vector functions
6. How to derive velocity from acceleration and position from velocity
7. Tangential and normal components of acceleration
8. Parametric surfaces and the role of grid curves in studying these surfaces
9. How the form and/or symmetry of a surface helps one in choosing a parametrization
10. Different parametrizations for surfaces

Homework for Wednesday
Homework Problems: WebAssign Assignment 5, WebAssign Test 1 Quiz
Test 1: Chapters 9-10

Reading the Text
Read Section 11.1 and answer the following questions

1. Why is the domain of the function in Example 4, $g(x,y) = \sqrt{9 - x^2 - y^2}$ shaped like a disk?

2. If f is a function of two variables and $f(3,-4) = -1$, give the coordinates of a point on the graph of f.

Math 213 Class 05: Unit Tangents and Unit Normals

1. Let $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 2t\,\mathbf{k}$.
   (a) Calculate the unit tangent vector $\mathbf{T}$.

   (b) Calculate the principle unit normal vector $\mathbf{N}$.

2. Curvature
   (a) What is the curvature of a straight line?

   (b) What is the curvature of a circle of radius 4?

3. If a particle moves at constant speed, what can be said about its acceleration? Consider the case where the particle moves in a straight line and the case where the particle moves along a curve. Talk about the normal and tangential components of acceleration.

4. Given the following position function $\mathbf{r}(t) = (2 + 3t + 3t^2)\mathbf{i} + (4t + 4t^2)\mathbf{j} + (-6\cos t)\mathbf{k}$ write $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ at $t = 0$. You do ***not*** need to do really complicated calculations.

5. A particle moves along the curve given by $\mathbf{r}(t) = t\,\mathbf{i} + \frac{\sqrt{6}}{2}t^2\,\mathbf{j} + t^3\,\mathbf{k}$.

a. Find the speed of the particle.

b. Find the tangential component of acceleration.

c. Find the normal component of acceleration.

d. Find the curvature of the curve.

Math 213 Class 05: Lost in Space

1. A space station is located at (0,0,0). A rocket is traveling along the path:

   $$\mathbf{r}(t) = (\ln t - 2)\mathbf{i} + \left(\frac{2}{t} + 2\right)\mathbf{j} + (3t - 9)\mathbf{k}$$

   If the rocket shuts off its engines at $t = 1$, will the rocket coast into the space station?

2. Planet A is moving on a path: $\mathbf{r}(t) = t^2\,\mathbf{i} + 8\,\mathbf{j} + (5t - 6)\mathbf{k}$

   Planet B is moving along: $\mathbf{s}(t) = (10 - 3t)\mathbf{i} + 2t^2\,\mathbf{j} + t^2\,\mathbf{k}$

   When will the 2 planets collide and what will be the position?

   Which planet will be moving faster when they collide?

   At what angle will they collide?

Olympic Torch

To open the 1992 Summer Olympics in Barcelona, bronze medalist archer Antonio Rebollo lit the Olympic torch with a flaming arrow. The event was photographed in a time exposure revealing the arrow's path of motion. The firing angle and speed of the arrow had to be precise – too large and fast and the arrow would fly too high over the cauldron to light it, too small and slow and the arrow would fly too low to hit the target. Rebollo had to do it right on his first try.

Suppose the cauldron was 70 feet high and 30 yards (90 feet) away from Antonio Rebollo. Further, suppose he wanted the flaming arrow to reach its maximum height exactly 4 feet above the center of the cauldron. He shoots from a height of 6 feet above ground level.

1. Draw a picture which diagrams the problem.
2. Express the maximum height in terms of the initial speed and firing angle.
3. Find the initial firing angle of the arrow.